Principles of Mathematics, Grade 10, Academic (MPM2D)

This course enables students to broaden their understanding of relationships and extend their problem-solving and algebraic skills through investigation, the effective use of technology, and abstract reasoning. Students will explore quadratic relations and their applications; solve and apply linear systems; verify properties of geometric figures using analytic geometry; and investigate the trigonometry of right and acute triangles. Students will reason mathematically and communicate their thinking as they solve multi-step problems.

Mathematical process expectations. The mathematical processes are to be integrated into student learning in all areas of this course.

Throughout this course, students will:

- **PROBLEM SOLVING** develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
- **REASONING AND PROVING** develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
- **REFLECTING** demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
- **SELECTING TOOLS AND COMPUTATIONAL STRATEGIES** select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
- **CONNECTING** make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
- **REPRESENTING** create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;
- **COMMUNICATING** communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Quadratic Relations of the Form $y = ax^2 + bx + c$

Overall Expectations

By the end of this course, students will:

- determine the basic properties of quadratic relations;
- relate transformations of the graph of $y = x^2$ to the algebraic representation $y = a(x - h)^2 + k$;
- solve quadratic equations and interpret the solutions with respect to the corresponding relations;
- solve problems involving quadratic relations.

Specific Expectations

Investigating the Basic Properties of Quadratic Relations

By the end of this course, students will:

– collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve of best fit, if appropriate, with or without the use of technology (*Sample problem:* Make a 1 m ramp that makes a 15° angle with the floor. Place a can 30 cm up the ramp. Record the time it takes for the can to roll to the bottom. Repeat by placing the can 40 cm, 50 cm, and 60 cm up the ramp, and so on. Graph the data and draw the curve of best fit.);

– determine, through investigation with and without the use of technology, that a quadratic relation of the form $y = ax^2 + bx + c$ $(a \neq 0)$ can be graphically represented as a parabola, and that the table of values yields a constant second difference (*Sample problem:* Graph the relation $y = x^2 - 4x$ by developing a table of values and plotting points. Observe the shape of the graph. Calculate first and second differences. Repeat for different quadratic relations. Describe your observations and make conclusions, using the appropriate terminology.);

– identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the y-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them;

– compare, through investigation using technology, the features of the graph of $y = x^2$ and the graph of $y = 2^x$, and determine the meaning of a negative exponent and of zero as an exponent (e.g., by examining patterns in a table of values for $y = 2^x$; by applying the exponent rules for multiplication and division).

Relating the Graph of $y = x^2$ and Its Transformations

By the end of this course, students will:

– identify, through investigation using technology, the effect on the graph of $y = x^2$ of transformations (i.e., translations, reflections in the x-axis, vertical stretches or compressions) by considering separately each parameter a, h, and k [i.e., investigate the effect on the graph of $y = x^2$ of a, h, and k in $y = x^2 + k$, $y = (x - h)^2$, and $y = ax^2$];

– explain the roles of a, h, and k in $y = a(x - h)^2 + k$, using the appropriate terminology to describe the transformations, and identify the vertex and the equation of the axis of symmetry;

- sketch, by hand, the graph of $y = a(x - h)^2 + k$ by applying transformations to the graph of $y = x^2$ [*Sample problem:* Sketch the graph of $y = -\frac{1}{2}(x - 3)^2 + 4$, and verify using technology.];
- determine the equation, in the form $y = a(x - h)^2 + k$, of a given graph of a parabola.

Solving Quadratic Equations

By the end of this course, students will:

- expand and simplify second-degree polynomial expressions [e.g., $(2x + 5)^2$, $(2x - y)(x + 3y)$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil) and strategies (e.g., patterning);
- factor polynomial expressions involving common factors, trinomials, and differences of squares [e.g., $2x^2 + 4x$, $2x - 2y + ax - ay$, $x^2 - x - 6$, $2a^2 + 11a + 5$, $4x^2 - 25$], using a variety of tools (e.g., concrete materials, computer algebra systems, paper and pencil) and strategies (e.g., patterning);
- determine, through investigation, and describe the connection between the factors of a quadratic expression and the x-intercepts (i.e., the zeros) of the graph of the corresponding quadratic relation, expressed in the form $y = a(x - r)(x - s)$;
- interpret real and non-real roots of quadratic equations, through investigation using graphing technology, and relate the roots to the x-intercepts of the corresponding relations;
- express $y = ax^2 + bx + c$ in the form $y = a(x - h)^2 + k$ by completing the square in situations involving no fractions, using a variety of tools (e.g. concrete materials, diagrams, paper and pencil);
- sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 - 2x - 8$ using intercepts and symmetry; sketching $y = 3x^2 - 12x + 1$ by completing the square and applying transformations; graphing $h = -4.9t^2 + 50t + 1.5$ using technology);
- explore the algebraic development of the quadratic formula (e.g., given the algebraic development, connect the steps to a numerical example; follow a demonstration of the algebraic development [student reproduction of the development of the general case is not required]);
- solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing) (*Sample problem:* Solve $x^2 + 10x + 16 = 0$ by factoring, and verify algebraically. Solve $x^2 + x - 4 = 0$ using the quadratic formula, and verify graphically using technology. Solve $-4.9t^2 + 50t + 1.5 = 0$ by graphing $h = -4.9t^2 + 50t + 1.5$ using technology.).

Solving Problems Involving Quadratic Relations

By the end of this course, students will:

- determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its defining equation (i.e., by applying algebraic techniques);
- solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Analytic Geometry

Overall Expectations

By the end of this course, students will:

- model and solve problems involving the intersection of two straight lines;
- solve problems using analytic geometry involving properties of lines and line segments;
- verify geometric properties of triangles and quadrilaterals, using analytic geometry.

Specific Expectations

Using Linear Systems to Solve Problems

By the end of this course, students will:

– solve systems of two linear equations involving two variables, using the algebraic method of substitution or elimination (*Sample problem:* Solve $y = \frac{1}{2}x - 5$, $3x + 2y = -2$ for x and y algebraically, and verify algebraically and graphically);

– solve problems that arise from realistic situations described in words or represented by linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method (*Sample problem:* The Robotics Club raised $5000 to build a robot for a future competition. The club invested part of the money in an account that paid 4% annual interest, and the rest in a government bond that paid 3.5% simple interest per year. After one year, the club earned a total of $190 in interest. How much was invested at each rate? Verify your result.).

Solving Problems Involving Properties of Line Segments

By the end of this course, students will:

– develop the formula for the midpoint of a line segment, and use this formula to solve problems (e.g., determine the coordinates of the midpoints of the sides of a triangle, given the coordinates of the vertices, and verify concretely or by using dynamic geometry software);

– develop the formula for the length of a line segment, and use this formula to solve problems (e.g., determine the lengths of the line segments joining the midpoints of the sides of a triangle, given the coordinates of the vertices of the triangle, and verify using dynamic geometry software);

– develop the equation for a circle with centre $(0, 0)$ and radius r, by applying the formula for the length of a line segment;

– determine the radius of a circle with centre $(0, 0)$, given its equation; write the equation of a circle with centre $(0, 0)$, given the radius; and sketch the circle, given the equation in the form $x^2 + y^2 = r^2$;

– solve problems involving the slope, length, and midpoint of a line segment (e.g., determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software).

Using Analytic Geometry to Verify Geometric Properties

By the end of this course, students will:

– determine, through investigation (e.g., using dynamic geometry software, by paper folding), some characteristics and properties of geometric figures (e.g., medians in a triangle, similar figures constructed on the sides of a right triangle);

- verify, using algebraic techniques and analytic geometry, some characteristics of geometric figures (e.g., verify that two lines are perpendicular, given the coordinates of two points on each line; verify, by determining side length, that a triangle is equilateral, given the coordinates of the vertices);
- plan and implement a multi-step strategy that uses analytic geometry and algebraic techniques to verify a geometric property (e.g., given the coordinates of the vertices of a triangle, verify that the line segment joining the midpoints of two sides of the triangle is parallel to the third side and half its length, and check using dynamic geometry software; given the coordinates of the vertices of a rectangle, verify that the diagonals of the rectangle bisect each other).

Trigonometry

Overall Expectations

By the end of this course, students will:

- use their knowledge of ratio and proportion to investigate similar triangles and solve problems related to similarity;
- solve problems involving right triangles, using the primary trigonometric ratios and the Pythagorean theorem;
- solve problems involving acute triangles, using the sine law and the cosine law.

Specific Expectations

Investigating Similarity and Solving Problems Involving Similar Triangles

By the end of this course, students will:

– verify, through investigation (e.g., using dynamic geometry software, concrete materials), the properties of similar triangles (e.g., given similar triangles, verify the equality of corresponding angles and the proportionality of corresponding sides);

– describe and compare the concepts of similarity and congruence;

– solve problems involving similar triangles in realistic situations (e.g., shadows, reflections, scale models, surveying) (*Sample problem:* Use a metre stick to determine the height of a tree, by means of the similar triangles formed by the tree, the metre stick, and their shadows.).

Solving Problems Involving the Trigonometry of Right Triangles

By the end of this course, students will:

– determine, through investigation (e.g., using dynamic geometry software, concrete materials), the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios (e.g., $\sin A = \frac{opposite}{hypotenuse}$);

– determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem;

– solve problems involving the measures of sides and angles in right triangles in real-life applications (e.g., in surveying, in navigating, in determining the height of an inaccessible object around the school), using the primary trigonometric ratios and the Pythagorean theorem.

Solving Problems Involving the Trigonometry of Acute Triangles

By the end of this course, students will:

– explore the development of the sine law within acute triangles (e.g., use dynamic geometry software to determine that the ratio of the side lengths equals the ratio of the sines of the opposite angles; follow the algebraic development of the sine law and identify the application of solving systems of equations [student reproduction of the development of the formula is not required]);

– explore the development of the cosine law within acute triangles (e.g., use dynamic geometry software to verify the cosine law; follow the algebraic development of the cosine law and identify its relationship to the Pythagorean theorem and the cosine ratio [student reproduction of the development of the formula is not required]);

– determine the measures of sides and angles in acute triangles, using the sine law and the cosine law (*Sample problem:* In triangle ABC, $\angle A = 35°$, $\angle B = 65°$, and AC = 18 cm. Determine BC. Check your result using dynamic geometry software.);

– solve problems involving the measures of sides and angles in acute triangles.